Topic: Graphs of inverse trig relations

Question: Imagine that you took the graph of the inverse relation for the function $\cos\theta$ (i.e., with the values of the function $\cos\theta$ on the horizontal axis and the measures of the corresponding angles θ on the vertical axis), and you drew a vertical line at $\cos\theta = \sqrt{3}/2$. What are the (measures of the) three smallest positive angles θ at which your vertical line intersects the graph of that inverse relation?

Answer choices:

A $\theta = \pi/6$, $\theta = 11\pi/6$, and $\theta = 13\pi/6$

B $\theta = \pi/3$, $\theta = 2\pi/3$, and $\theta = 4\pi/3$

C $\theta = \pi/6$, $\theta = 5\pi/6$, and $\theta = 7\pi/6$

D $\theta = \pi/3$, $\theta = 5\pi/3$, and $\theta = 7\pi/3$

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Solution: A

The first (vertical) interval above the horizontal axis corresponds to angles θ in the interval $[0,2\pi)$. Recall that for any y in the interval $(-1,1)$, there are exactly two angles θ at which the value of the cosine function is equal to y. Thus the two smallest positive angles θ whose cosine is equal to $\sqrt{3}/2$ are in the interval $[0,2\pi)$, and the third smallest positive angle θ whose cosine is equal to $\sqrt{3}/2$ is the smaller of the two angles in the interval $[2\pi,4\pi)$.

Recall that

$$\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$$

Since $\pi/6$ is an acute angle, the smaller of the two angles θ in the interval $[0,2\pi)$ whose cosine is equal to $\sqrt{3}/2$ is $\pi/6$. The (measure of the) other angle θ in the interval $[0,2\pi)$ whose cosine is equal to $\sqrt{3}/2$ is

$$2\pi-\frac{\pi}{6}=\frac{6(2\pi)-1(\pi)}{6}=\frac{12\pi-\pi}{6}=\frac{11\pi}{6}$$

To verify this, we can apply the difference angle for cosine:

$$\cos\left(2\pi-\frac{\pi}{6}\right)$$

$$\cos(2\pi)\cos\frac{\pi}{6}+\sin(2\pi)\sin\frac{\pi}{6}$$

$$(1)\left(\frac{\sqrt{3}}{2}\right)+(0)\sin\frac{\pi}{6}$$

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$$\frac{\sqrt{3}}{2}$$

The (measure of the) smaller of the two angles θ in the interval $[2\pi,4\pi)$ whose cosine is equal to $\sqrt{3}/2$ exceeds $\pi/6$ by 2π, so its measure is

$$2\pi + \frac{\pi}{6} = \frac{6(2\pi) + 1(\pi)}{6} = \frac{12\pi + \pi}{6} = \frac{13\pi}{6}$$

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Topic: Graphs of inverse trig relations

Question: Which of the following describes the graph of the inverse relation for the function $1.5 \sin \theta$?

Answer choices:

A The graph of the inverse relation for the function $1.5 \sin \theta$ spans an interval of length 2 on the horizontal axis, and the smallest height for which the size and shape of the graph are identical on consecutive vertical intervals (of angle measure) is 6π.

B The graph of the inverse relation for the function $1.5 \sin \theta$ spans an interval of length 2 on the horizontal axis, and the smallest height for which the size and shape of the graph are identical on consecutive vertical intervals (of angle measure) is 3π.

C The graph of the inverse relation for the function $1.5 \sin \theta$ spans an interval of length 1.5 on the horizontal axis, and the smallest height for which the size and shape of the graph are identical on consecutive vertical intervals (of angle measure) is 2π.

D The graph of the inverse relation for the function $1.5 \sin \theta$ spans an interval of length 3 on the horizontal axis, and the smallest height for which the size and shape of the graph are identical on consecutive vertical intervals (of angle measure) is 2π.

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Solution: D

The amplitude of the function $1.5 \sin \theta$ is 1.5. Recall that the amplitude of a sine or cosine function is equal to one-half the difference between the maximum and minimum values of that function. Since 1.5 is one-half of 3, the difference between the maximum and minimum values of the function $1.5 \sin \theta$ is 3, so the graph of the inverse relation for the function $1.5 \sin \theta$ spans an interval of length 3 on the horizontal axis.

The length of the interval of the horizontal axis which is spanned by the graph of the inverse relation for $1.5 \sin \theta$ has no effect on the smallest height for which the size and shape of the graph are identical on consecutive vertical intervals (of angle measure), so the smallest height for which the size and shape of the graph of the inverse relation for $1.5 \sin \theta$ are identical on consecutive vertical intervals is the same as the smallest height for the basic sine function (namely, 2π).

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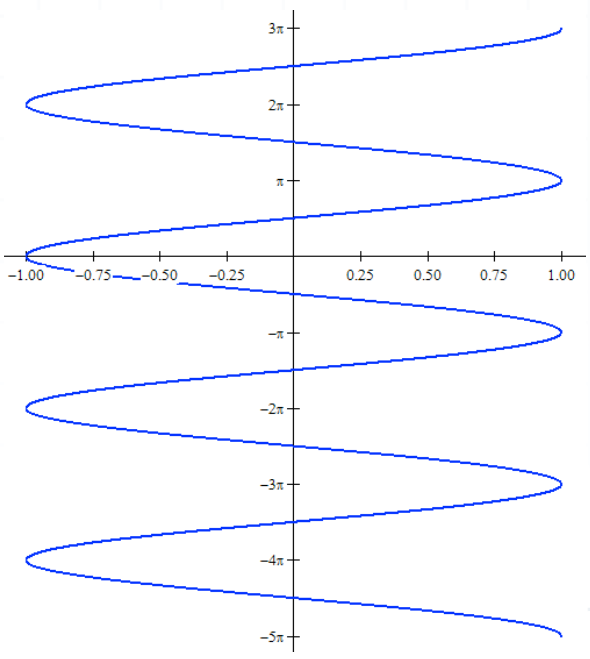
Topic: Graphs of inverse trig relations

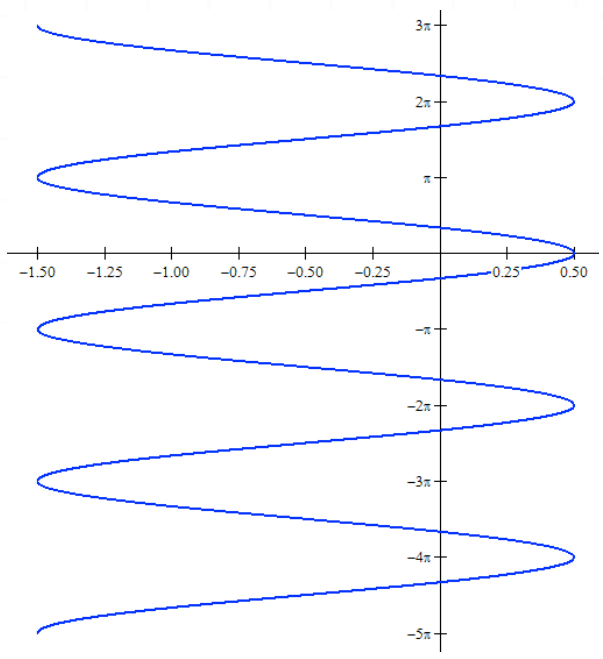
Question: Which of the following is the part of the graph of the inverse relation for the function $\cos\theta + 1$ that corresponds to angles θ in the interval $[-5\pi, 3\pi]$?

Answer choices:

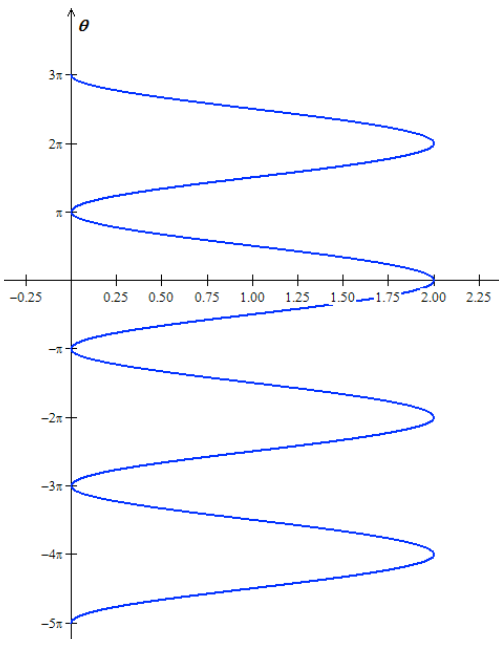
A



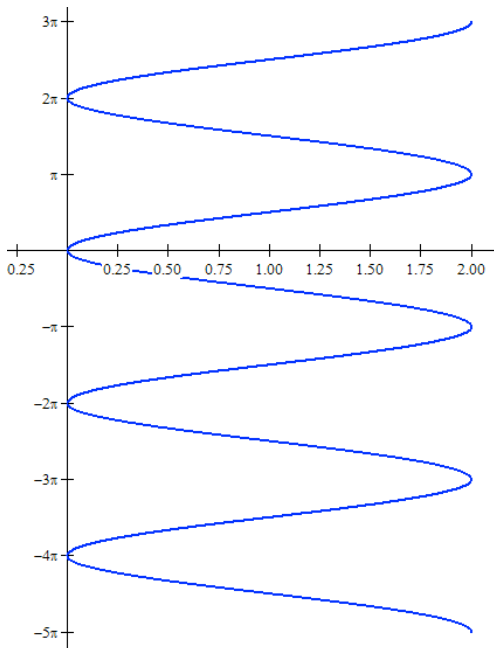
C



B



D



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Solution: B

The values of the function $\cos\theta + 1$ are shifted upwards by 1 unit (note that the graph of $\cos\theta + 1$ is shifted vertically by 1 unit upwards, compared to the graph of the function $\cos\theta$), so they range over the interval $[0,2]$ instead of over the interval $[-1,1]$. This means that the graph of the inverse relation for the function $\cos\theta + 1$ spans the interval of the horizontal axis whose left endpoint is 0 and whose right endpoint is 2.

Notice that the interval of the horizontal axis which is spanned by the graphs given in answer choices B and D is indeed $[0,2]$, so at this point we have to consider both of those graphs as possible correct answers.

We can immediately rule out answer choices A and C, however, because the interval of the horizontal axis which is spanned by the graph given in answer choice A is $[-1,1]$ (not $[0,2]$), and the interval of the horizontal axis which is spanned by the graph given in answer choice C is $[-1.5,0.5]$ (again, not $[0,2]$).

For a different way to rule out answer choices A and C, recall that $\cos(n\pi) = -1$ if n is an odd integer and $\cos(n\pi) = 1$ if n is an even integer. Thus

$$n \text{ odd} \implies \cos(n\pi) + 1 = -1 + 1 = 0$$

and

$$n \text{ even} \implies \cos(n\pi) + 1 = 1 + 1 = 2$$

On the graph given in answer choice A, the angles θ that correspond to 0 on the horizontal scale are $5\pi/2$, $3\pi/2$, $\pi/2$, $-\pi/2$, $-3\pi/2$, $-5\pi/2$, $-7\pi/2$, and

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$-9\pi/2$ (that is, they aren't odd multiples of π), and there are no angles that correspond to 2 on the horizontal scale.

On the graph given in answer choice C, the angles θ that correspond to 0 on the horizontal scale aren't integer multiples of π, and there are no angles that correspond to 2 on the horizontal scale.

On the graph given in answer choice D, the angles θ that correspond to 0 on the horizontal scale are 2π, 0, -2π, and -4π (that is, even multiples of π, not odd multiples of π), and the angles that correspond to 2 on the horizontal scale are 3π, π, $-\pi$, -3π, and -5π (that is, odd multiples of π, not even multiples of π). Thus we can also rule out answer choice D.

Notice that in the graph given in answer choice B, the angles θ that correspond to 0 on the horizontal scale are the odd multiples of π, and the angles that correspond to 2 on the horizontal scale are the even multiples of π.

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